Symmetry I: A basic concept in crystals and everyday life

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Teaching the basic concepts of symmetry is of utmost importance not only in crystallography, but also in numerous other disciplines such as chemistry, biology, etc. and even in apparently more remote fields such as arts. It is the aim of the present chapter to increase students' understanding of symmetry as an inherently present phenomenon in everyday life. During this course the strict ideas of geometric symmetry, e.g. in crystals, are opened up towards a more general idea of "symmetry = repetition by a certain rule", leaving behind the classical coordinates of space, identical motifs, and dimensions of translation.

The topic is subdivided into six short chapters given below:
1) Review of symmetry and its purpose in crystallography, biology, tools, etc.
2) The ten two-dimensional crystallographic point groups
3) The plane groups/patterns
4) The frieze/strip groups
5) Symmetry beyond the coordinates of space; homeometric symmetry / similarity
6) Three-dimensional point groups

1) Symmetric objects are frequently recognized by their appealing shapes, composed by the repetition of certain motifs by symmetry operations/elements such as rotation/rotation axes, reflections/mirror planes, and inversion/center of symmetry.

a) b) c)

Figure 1. a) Tetragonal crystal on the cover page of Mitt. Österr. Miner. Ges. b) The Orthodox Church in Bar, Montenegro. c) The common pattern of a brick wall (*c*2*mm*).

This esthetic aspect is observed e.g. in crystals (Fig. 1a), architecture (Fig. 1b) and sport disciplines. On the other hand, symmetry has a certain function in biology, e.g. for directed movement and selective reproduction, and in the construction of technical tools. In addition, repetition by translation provides the advantage to build up a whole big object from small equal pieces in an ordered way as, e.g. a wall is built up from bricks (Fig. 1c). The extreme of this concept is observed in crystals where billions of atoms are neatly arranged in a lattice.

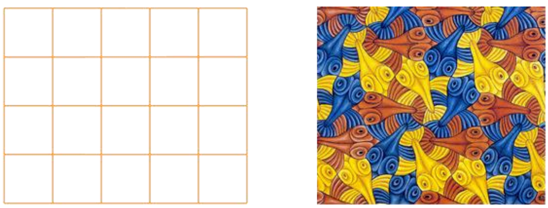
2) The most simple and instructive way to explore symmetry is to start with the ten two-dimensional crystallographic point groups, which are obtained by five possible rotations alone (1, 2, 3, 4, 6) or in combination with reflections at mirror planes (*m*, 2*mm*, 3*m*, 4*mm*, 6*mm*). Note that the axes are restricted to those which are in agreement with a crystallographic lattice; else the number of groups would be infinite!

a) b) c)

Figure 2. Examples of 2-dim. crystallographic point groups. a) 2*mm* b) 3*m* c) 4*mm*.

These 10 groups can be exercised in numerous ways, e.g. using flags (Fig. 2a), mystic symbols, car logos (Fig. 2b), traffic signs (Fig. 2c), geometric forms, and even alpha-numeric characters. Regarding the latter, e.g. H, 0, I belong to point group 2*mm* with a horizontal and vertical mirror plane plus a 2-fold axis in the center. Eventually, an animated PowerPoint application has been developed (Libowitzky 2012), where students can strengthen their ability in symmetry identification.

3) With the addition of translation and the new symmetry element of the glide plane, the 17 plane groups/patterns with their two-dimensional lattices ("Two-dimensional space groups", e.g. Hahn 1996) are composed. Besides of their importance as projections of the crystallographic space groups, they are frequently observed in tile patterns and parquet floors, wall papers, ties, clothes, etc. and thus are easily susceptible to students' investigations. As an example the most common array of square tiles (Fig. 3a) is assigned to the plane group *p*4*mm*. A more challenging



a) b)

Figure 3. Plane groups. a) Pattern of square tiles: *p*4*mm*. b) M.C. Escher's pattern "Fish" (Buseck 1997): *p*2 with color distinction or *p*6 without color distinction.

exercise is the assignment of plane groups to the famous pictures of M.C. Escher (Locher 1984, Buseck 1997). While his pattern "Unicorn" provides an instructive example of the glide plane in group *pg*, his pattern "Fish" belongs to *p*2 or *p*6 depending on color distinction (Fig. 3b). Finally, his pattern "Shells and Starfish" is an excellent example for local high symmetry broken by a disordered substructure, commonly found in the real structure of crystals.

4) If two-dimensional groups are reduced to only one direction of translation (in the 2nd direction they behave like point groups), the so called 7 frieze or strip groups result. These are commonly found in the form of ornaments in architecture, on ancient jugs and pots, rims of clothes, etc. (Hargittai & Hargittai 1994). They may comprise (alone or in combination) a 2-fold axis, mirror planes (both directions possible), a glide plane (only along the strip elongation, i.e. direction of translation). One of the most common groups is *p*2*mg*, found e.g. in the plot of a sine wave or a zig-zag line (Fig. 4a,b), or in a picture of the battlements of a castle wall (Fig. 4c).

a) b) c)

Figure 4. Frieze/strip group *p*2*mg*. a) Sine plot. b) Zig-zag line. c) Square function.

5) Even better, if the coordinates of space are left behind, and the direction of translation is replaced by a time coordinate (everything is repeated with strict time intervals, i.e. with a rhythm), everyday life provides almost infinite examples of the frieze group *p*2*mg* such as the blinking of a traffic light, an electromagnetic wave, and numerous beats in modern pop music. The latter is only a starting point to the wide field of symmetry in music, where translation in time is a common phenomenon (e.g. JS Bach's passacaglia in c-minor), and even higher symmetry is encountered here and there (Fig. 5a; Preisinger 1980). The frieze groups, however, are also well suited to demonstrate a further step towards generalization of symmetry. Instead of a mere repetition of an identical motif/object by a symmetry operation, a similar one (e.g. with a different size) is reproduced (Fig. 5b). The so called similarity symmetry (homeometric symmetry) is frequently found in nature (Fig. 5b,c), e.g. in snail shells, twigs, etc., but also in mathematical sequences and series, and even in a few pictures of MC Escher (Locher 1984). Eventually, the concept of symmetry can be further generalized towards an esthetic relation of the single parts to the bulk, as it is verified e.g. in the "sectio aurea", the golden section.

a) b) c)

Figure 5. a) Local symmetry 2*mm* in coordinates of time (horizontal) and frequency / sound in a music example (Preisinger 1980). b) Homeometric symmetry / similarity in a frieze group with only one similarity glide plane, and c) in a Nautilus shell.

6) The three-dimensional point groups are limited in crystals to 32 so called "crystal classes", as their symmetry axes (only 1, 2, 3, 4, 6, and $\bar{4}$) must be in agreement with the internal crystal lattice. Thus, the outer shape of a crystal can be readily described, with symmetry-equivalent faces building up characteristic forms (e.g. octahedron, prism). Even better, the symmetry imposes certain constraints on the physical properties of a crystal (Putnis 1992. Two common examples are that cubic crystals are optically isotropic, as the tensor ellipsoid of the refractive index is reduced to a sphere, or that a center of symmetry excludes the piezo-electric effect, as polar charging is impossible. On the other hand, the

number of groups is infinite, if not constrained by a lattice. For example, symmetries with 5-fold axes are encountered in certain molecules, in biology (e.g. Rosacea, Echinodermata) and technics (e.g. PET bottles, car wheels). Even there, symmetry affects mechanical stability in the latter, or constrains number and type of possible vibrations in molecular spectroscopy. As a students' exercise, objects from everyday life are inspected (e.g. symmetry *mm*2 of an open shoe box), and finally a number of groups are composed by our own hands, feet, and bodies.

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